THE COMPENDIUM

JOURNAL OF THE NORTH AMERICAN SUNDIAL SOCIETY

VOLUME 31 ISSUE 3 - SEPTEMBER 2024

A MONUMENTAL DIAL AS A SHADOW TERMINATOR

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This article was originally published in Orologi Solari n. 32, Dec. 2023, as *Un quadrante monumentale a terminatore d'ombra*.

Introduction

Once again a project came to fruition thanks to Aurelio Pantanali´s assiduous initiative: his contribution is indeed fundamental both for the perfect success of the Sundial Festival, which takes place every year in Aiello, and for the creation of many sundials. It is precisely one of these projects that we want to tell you about here.

In October 2020, Pantanali contacted Gian Casalegno to ask for his collaboration in a project to place a shadow terminator sundial on a water tank in the town of Borbiago, in the municipality of Mira in the province of Veneto.

Fig. 1. The water tank in Borbiago in Mira.

From this first contact, a project was born that has only recently been completed, and which subsequently also involved Giuseppe De Donà.

The Borbiago tower, built between 1958 and 1960, is 32 meters high and guarantees the distribution of water in the area. The tank, made of reinforced concrete, is located 25 m from the ground, is 5 m high, has a diameter of 12.7 m and is capable of containing 450 m^3 of drinking water, used as a storage and reserve for the town of Borbiago.

The cylindrical tank is the surface where the sundial was to be drawn.

The Shadow Terminator Sundial

This type of sundial is rather unusual and consequently hardly known by the public and perhaps even by some gnomonists.

Yet it has an undeniable advantage over traditional sundials: it does not need a style, and it is the same solar shadow that moves over time on the surface of the object (a cylinder in our case) which allows us to read the time on the hour lines drawn on the surface.

Unfortunately, this prerogative is accompanied by an equally undeniable disadvantage: it is not easy to identify the precise edge of the shadow on the surface of the object. This difficulty causes an imprecision in the identification of the time, of the order of approximately ±15 minutes, and is highly dependant on the color of the surface and the resulting shadow-light contrast.

However, as explained below, the project is not complicated at all.

In the case of a vertical cylinder, the shadow created by sunlight extends for ±90º from the direction of the rays. It is therefore sufficient to calculate the solar azimuth for each day of the year and to identify the points positioned at ±90º with respect to the Sun as the extremes of the shadow. The union of the points corresponding to the same time on different dates creates the desired hour line.

However, we need to choose which terminator is to be used: the Eastern one or the Western one[1]. Inserting both is problematic as the hour lines would intersect each other in the south direction. The Borbiago sundial makes use of the Eastern one, therefore with value A – 90°.

Let's call A the azimuth of the Sun, H the hour angle, φ the latitude of the place, δ the solar declination; we thus have [1]:

[1] Note that the hour lines are the same whether you choose the East or the West terminator: they are simply shifted relative to each other by 180º.

$$A = \tan^{-1}\left(\frac{\sin(H)}{\sin(\varphi)\ \cos(H) - \cos(\varphi)\tan(\delta)}\right) \qquad (1)$$

The time is read at the point of intersection between the vertical line separating light and shadow (the shadow terminator) and the hour and date lines. In the example in Figure 2, the position at 11 a.m. on December 1st is calculated.

Of course, the points obtained with the expression (1) must be accepted, and used to compose the graph of the hour lines, only if the Sun height h is greater than 0°, where h is calculated from the relation:

$$h = \sin^{-1}[\sin(\varphi)\sin(\delta) + \cos(\varphi)\cos(\delta)\cos(H)] \qquad (2)$$

<u>Hour line computation</u>

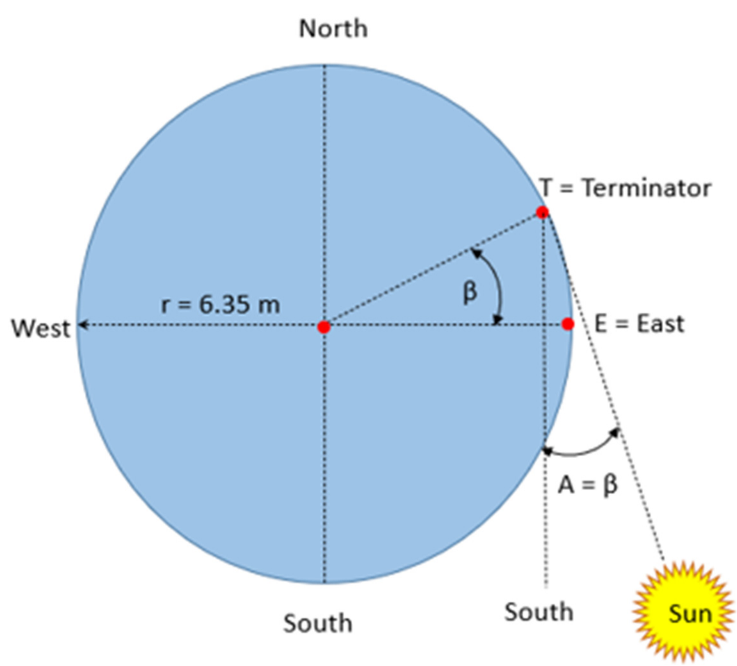

Fig. 2. Calculating the position of the terminator.

In a circle of radius r, the length of the arc ET that subtends the angle β is given by:

$$ET = \frac{2\pi r\beta}{360°}$$

For example, on December 1st 2023, at 11:00 local time, in Borbiago the azimuth of the Sun is:

$$A = \beta = -14° \, 57'21.5''$$

Hence for the water tank with r = 6.35 m:

$$ET = \frac{2\pi \, 6.35 \, (-14.956°)}{360°} = -1.657 \, m$$

Starting from the vertical indication of 12 o'clock located in the East, the calendar line of December 1st intersects the 11 o'clock curve at a distance of 1,657 m measured from East to North.

The hour angle H can be evaluated to include the difference in longitude (to indicate the time in the time zone) and the Equation of Time (to indicate the average time). For the Borbiago sundial the choice was to indicate the true local time and therefore simply:

$$H = 15° \, (t - 12) \qquad (3)$$

where t is the desired local time.

Figure 3 shows the resulting hour plot, which is the location on the side surface of the cylinder where the hour lines should be drawn.

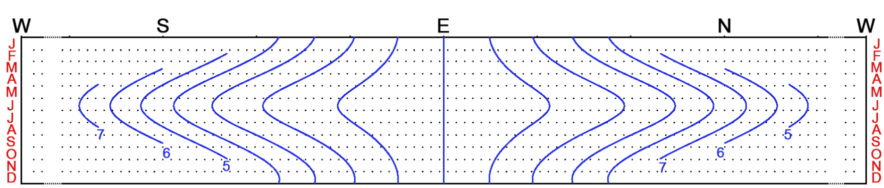

Fig. 3. Resulting hour plot.

Figure 4 shows the shadow simulation at 9 am local time on June 21st.

In order to correctly trace the hour lines on the tower as shown in Figure 3, it is necessary to accurately identify the East direction at which the 12 o'clock time line will coincide: a more complicated task than what we are used to when we measure the declination of a wall.

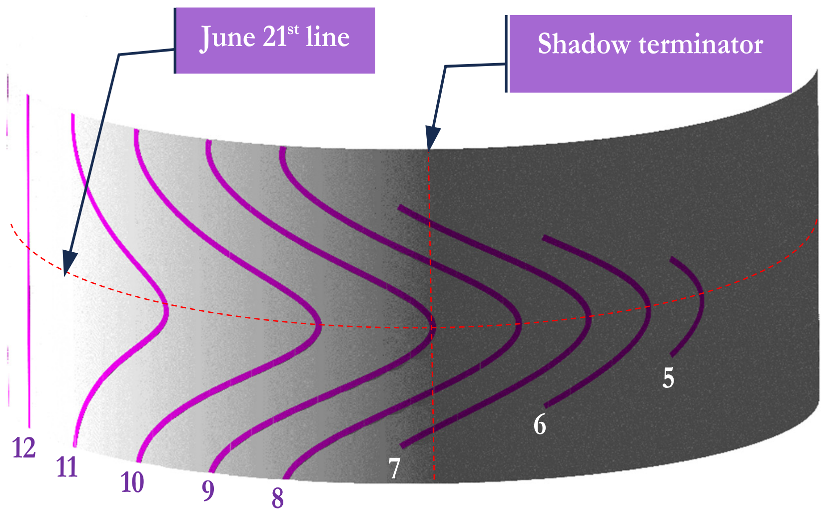

Fig. 4. Shadow simulation at 9 am local time on June 21st.

It is advisable to carry out this operation on the ground so that, when the decorators go up, the position of the 12 o'clock time line can be quickly and precisely indicated to them even without the aid of the sun.

Determination of the East direction

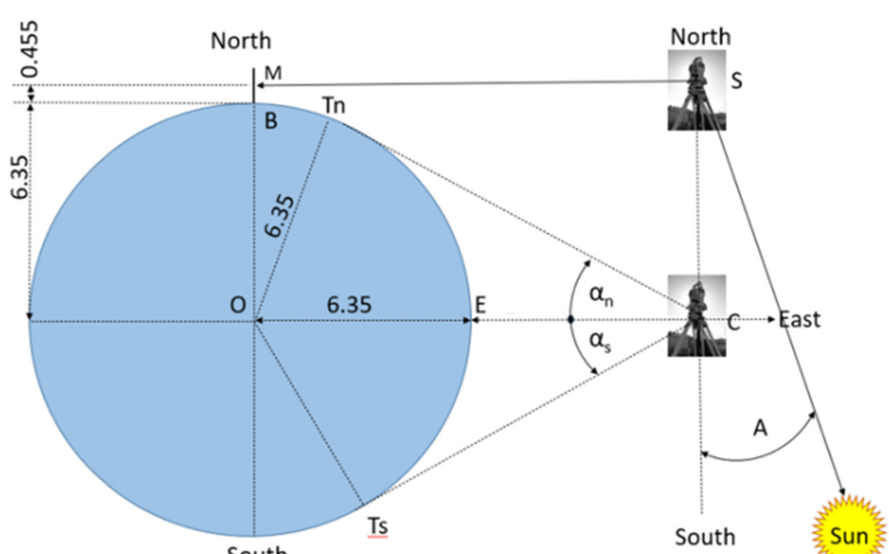

Fig. 5. Measurement scheme.

On September 7th, Giuseppe De Donà, Aurelio Pantanali and Gregorio Barp performed, with a Wild T2 theodolite [2], the operations that are schematized in Figure 5.

In a paved area located to the east of the piezometric tower, a point **S** was identified at a distance great enough to be able to operate comfortably with the telescope of the instrument aiming upwards to the tank.

The reference in **S** (a steel nail planted in the asphalt) was placed slightly north of the edge **B** identified on the East-West northern tangent to the cylindrical base (which has the same radius as the water tank).

With the telescope of the instrument equipped with a solar filter, the Sun was centered in the grating while simultaneously capturing a screenshot of the *Sol et Umbra App* [3] (Figs. 6, 7 and 8).

Fig. 6. Sun collimation by De Donà and screenshot by Gregorio.

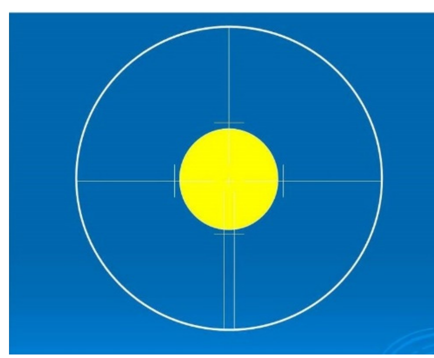

Fig. 7. The Sun centered in the T2 lattice.

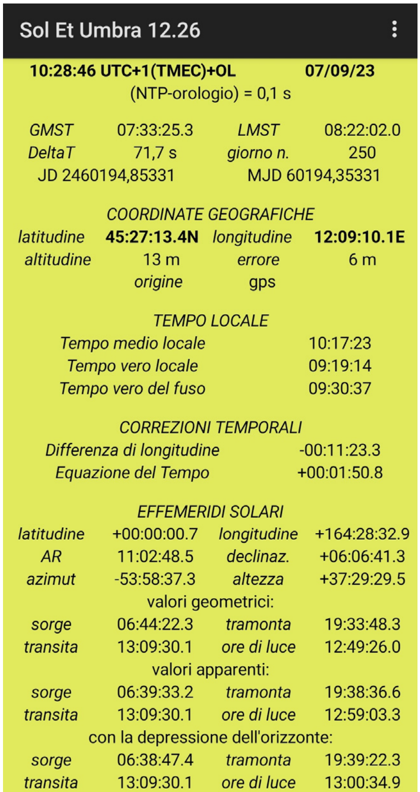

Fig. 8. Screenshot from 'Sol et Umbra'.

Data provided by Sol et Umbra, useful for this work, are the following:

Latitude (φ): 45° 27′ 13.4″ N

Longitude: 12° 09′ 10.1″ E

Central European Time:
09h 28m 46s

Local Apparent Time (t):
09h 19m 14s

Sun Azimuth (A):
−53° 58 ′ 37.3″

Sun Height (h): 37° 29′ 29.5″

Sun Declination (δ):
06° 06′ 41.3″

The azimuth A and the height h, as calculated by the app, are identical to the values obtained by means of a different software program [4] using φ, δ and H with equations (1), (2) and (3).

To ensure the correct positioning of the instrument, we verified that the height of the Sun read on the zenith vernier (90° − h) was congruent with the theoretical value from *Sol et Umbra*. Subsequently, with the alidade blocked, the azimuth was calibrated at:

$$-53^\circ\,58'\,37.3'' + 360^\circ\,00'\,00'' = 306^\circ\,01'\,22.7''\ ^{2}$$

[2] In the Wild T2 instrument, all angles are measured as centesimal degrees (gon) where the round angle is 400 gon. For this paper, all the readings have been transformed into the traditional sexagesimal system.

Then, once the alidade was unlocked, the instrument was rotated to the West direction (Azimuth A = 90°), where, with the instrument's telescope pointed at a meter rule placed in **B** orthogonal to the East-West direction, the MB measurement was read as 45.5 cm.

The design radius of the cylinder is 6.35 m, therefore the distance **MO** is 6.35 + 0.455 = 6.805 m, so by rotating the instrument towards the South (A = 0°) it was possible to place at **C**, at a distance **SC = MO**, a steel nail exactly East of the center **W** of the tank.

Positioning at **C**, the point **S** was targeted by calibrating the azimuth vernier at 180°. By rotating the telescope once again to the West (A = 90°), it was possible to trace the reference **E** and measure the distance **CE** equal to 30.35 m (with a Hilti PD4 distance meter), from which **CO** = 30.35 + 6.35 = 36.70 m.

Considering the right triangle **O$\mathbf{T_n}$C,** where **CO** and **$\mathbf{T_n}$O** are known, the angle α_n (equal to α_s) can be obtained:

$$\alpha_n = \sin^{-1}\frac{OTn}{OC} = 9.963°$$

Still from **C**, collimating **$\mathbf{T_n}$** and **$\mathbf{T_s}$** with the telescope, the width of the angle ($\alpha_n + \alpha_s$) was read as 19.93°, so, although with a very slight (and therefore negligible) difference between α_n and α_s, it was possible to confirm the radius of the tank and the correct location of the benchmark **C** as the point from which to mark the vertical East line, i.e. the origin of the sundial layout.

Before dismantling the instrument, the telescope was rotated onto the Church of Borbiago, detecting the azimuth of the bell tower's cross as equal to 210.495° (Fig. 9), as a reference point from **C** for the executive operations which were carried out on the morning of September 15th.

Fig. 9. The cross of the bell tower in Borbiago provided a reference for the layout.

Tracing the sundial lines

After decoration of the column, the tracing of hour lines on the cylindrical tank began on September 15th, started by Giacomo Pantanali and then continued by the decorators from the Alessandro Ferri (Dodo) company.

The work was carried out with the aid of two truck-mounted crane baskets. Figures 10 and 11 show some phases of the tracing and decoration work on the sundial.

Fig. 10. Crane-mounted 'cherry-picker' work platform.

Fig. 11. The crane's telescopic arm extended.

To facilitate the operations and to trace the hour lines in a faster and more precise way, Giacomo and Aurelio Pantanali had prepared curved metric templates with a radius equal to the external radius of the tank (Figs. 12 and 13).

The photos in Figures 14 and 15, taken on November 29th, highlight how the precision of the sundial is as expected, i.e. ±15 minutes (or perhaps even a little better), therefore quite satisfactory.

Fig. 12. The arched wooden metric templates with the radius of the tank, used to trace the hour lines.

Top left: Fig. 13. Alessandro Ferri, Aurelio and Giacomo Pantanali pose with the metric templates.

Bottom left: Fig. 14.
Bottom right: Fig. 15.

Two images taken respectively at 11:00 and 13:55 on November 29th, the day on which civil time coincides with local time in Borbiago; the reading must be taken at the height of the lower edge of the lower windows.

The work was completed in September 2023. On November 29th the result was presented in Borbiago by Andrea Razzini, director of Veritas, in the presence of Marco Dori, mayor of Mira, and his colleague Tiziano Baggio, mayor of Mirano.

The short ceremony took place at the foot of the tower where explanatory panels were posted.

The sundial was illustrated by the artist Alessandro Ferri and the gnomonist Giuseppe De Donà (Fig. 16). Figures 17 and 18 show two images of the finished work.

Fig. 16. November 29th, Alessandro Ferri is describing the Borbiago sundial to participants. To the left are Giuseppe De Donà and Marco Dori, mayor of Mira.

Fig. 17. The water tower seen from the south with the bell tower of the Borbiago church.

Fig. 18. The attractive decoration of the tank sundial and of the supporting tower, as seen from the square to the east.

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